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TECHNICAL NOTE

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ANALYSIS OF THE TRANSIENT RADIATION HEAT TRANSFER OF AN UNCOOLED ROCKET ENGINE OPERATING OUTSIDE EARTH'S ATMOSPHERE

By William H. Robbins

Lewis Research Center
Cleveland, Ohio

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SUMMARY

The heat-transfer characteristics of a body in space are analyzed during the period when the temperature of the body varies with time. Relations for the cooling rate of a body in space are derived and presented. The rate of radiant heat dissipation (reflected by the temperature) is determined to be a function of material properties of the body, the geometry of the body, and the temperature of both the body and the effective surrounding temperature.

The application of the results of the analysis is directed toward the uncooled (heat sink) high-chamber-pressure rocket engine. Cooling times from the temperature at burnout to a specified final temperature are presented for a range of thrusts and running times. In general, the cooling time increases almost linearly with running time. At relatively low values of thrust, the cooling time increases with thrust. In contrast, at higher thrust levels, the cooling time decreases significantly. In general, rapid intermittent operation of uncooled rocket engines in space (periods of 1 hr or less) is impractical.

INTRODUCTION

Considerable research effort has been directed toward the design and development of manned vehicles to operate outside Earth's atmosphere. Since maneuverability is a requirement of these vehicles, propulsive power from rocket motors must be provided. The rocket engines can be utilized, for example, for navigational course correction during a specified mission or maintaining or adjusting a satellite orbit.

Since the combustion temperature for most chemical rocket propellants exceeds the material limits, some form of cooling or heat dissipation from the rocket walls must be provided. In the past, the cooling techniques have fallen into two general categories. Either the rocket was regeneratively cooled (the rocket propellants were utilized to absorb the high heat release of combustion) or the rocket walls were designed to act as a heat sink. Both methods have been successful, although the heat-sink method implies a limited running time. Another engine cooling scheme is now receiving considerable attention - cooling by means of radiation. Steady-state radiation cooling appears to be a promising technique in engines with low heat-flux rates (ref. 1). With most chemical propellants, however, low heat-flux rates can be achieved only at very low chamber pressures (5 to 10 lb/sq in. abs). In addition to the practical problems of ignition under this condition, these devices are associated with the fundamental problems of low thrust for a given engine size and low specific impulse unless the nozzle is very large.

In space missions where rapid acceleration is desirable, high thrust becomes a requirement of the engine. The analysis in this report is directed toward this application. Since high-thrust devices are associated with high heat-flux rates, instantaneous and continuous heat dissipation by means of radiation becomes impractical, and the regeneratively cooled or heat-sink engine must be utilized. Since good propellant storage characteristics become a prime requirement, solid propellants and many liquid propellants with poor cooling characteristics become of interest. Therefore, in many cases the design of the engine is limited to the heat-sink type, and intermittent rather than continuous operation is necessary. The design of heat-sink rockets for the application is associated with the problem of heat conduction through the walls during the firing time and the dissipation of heat by means of radiation during the cooling period. In both regimes the temperature of the powerplant varies continuously with time (non-steady-state operation). The analysis is primarily concerned with the cooling cycle to determine the feasibility and effectiveness of radiation cooling over a range of thrusts and running time.

A time-temperature cycle for the analysis is specified arbitrarily. It is assumed that the powerplant is heated (by convection and conduction) from an initial temperature to a final temperature at the end of burning. The rocket is then cooled to the initial temperature by radiation of heat to space. The second phase of the cycle (radiation heat transfer) is discussed in the ANALYSIS section of this report. A discussion of the heating cycle is reserved for the section entitled APPLICATION OF ANALYSIS.

ANALYSIS

This section is concerned with the cooling characteristics of a body in space. Both the approach to the analysis problem and a discussion of the analysis results are presented.

Analysis Approach

The analysis is directed toward a body (removed from Earth's atmosphere) that is dissipating heat by means of radiation. It is assumed that the temperature of the body is higher than the temperature of its surroundings. The analysis is limited to the period in which the temperature of the body is continuously varying with time. Therefore, the analysis would be applicable, for example, to a space vehicle cooling from an elevated temperature to an equilibrium temperature after it has been heated for a period of time by an internal heat source such as a reactor or chemical rocket.

The surroundings of a body are associated with a hypothetical temperature. The effective surrounding temperature is a function of the amount of radiant energy received from the sun and all other celestial bodies; and the quantity of radiant energy received by the body is, in turn, dependent upon the orientation of the body with respect to the sun and other celestial bodies. Values of surrounding temperature range from 700° R for a body in a close Earth-satellite orbit exposed to direct sunlight (ref. 2) to approximately absolute zero for vehicles far removed from the planets and not exposed to sunlight. The surrounding temperature is referred to as "sink temperature" herein. The sink temperature is not evaluated, since it depends upon the specific mission of the vehicle. The analysis is presented for an arbitrarily selected range of sink temperature of 700°, 400°, and 0° R.

It is assumed that none of the emitted energy from the body returns to the body; in other words, the radiation emitted from a surface area element never impinges on any other element of the body. The validity of this assumption depends upon the body geometry. For the configurations considered in this analysis (rocket engine), the assumption is felt to be valid.

The surface of the body is "gray." A gray surface is characterized by constant absorptivity for radiation of all wavelengths. In addition, the emissivity is equal to the absorptivity of the surface. This equality does not generally exist for most materials (ref. 3). The assumption is made because the absorption characteristics of surfaces are not generally available.

It is assumed that the conduction of heat from within the vehicle to the radiating surface is instantaneous during the cooling period. This implies that no temperature gradients exist within the body (infinite conductivity). This assumption is made frequently in radiation analyses and is felt to be valid.

Results of Analysis

The assumptions listed previously in conjunction with the energy equation and radiation equation were utilized to derive relations for the cooling time of a body in space. The energy equation can be written as follows:

$$\frac{dQ_{b,n}}{dt} = -W_b c \frac{dT_b}{dt} \tag{1}$$

(Symbols are defined in appendix A.) The radiation equation is expressed as

$$\frac{dQ_{b,n}}{dt} = \epsilon_b \sigma S_b (T_b^4 - T_s^4) \tag{2}$$

where T_s is the effective temperature of space or sink temperature.

The simultaneous solution of equations (1) and (2), with the boundary condition at time equal to zero and the initial temperature equal to $T_{b,0}$, results in the following relation (details of the derivation are presented in appendix B):

$$t = \frac{\rho c V}{\epsilon \sigma S_b T_s^3}\left[\frac{1}{4}\ln\left|\frac{T_b + T_s}{T_b - T_s}\right| + \frac{1}{2}\tan^{-1}\left(\frac{T_b}{T_s}\right) - \frac{1}{4}\ln\left|\frac{T_{b,0} + T_s}{T_{b,0} - T_s}\right| - \frac{1}{2}\tan^{-1}\left(\frac{T_{b,0}}{T_s}\right)\right] \tag{3}$$

Equation (3) expresses the cooling time of a body in terms of body and space temperatures, the material properties (ρ, c, ϵ), and the body geometry (V/S). When the initial temperature $T_{b,0}$ approaches infinity, equation (3) becomes

$$t = \frac{\rho c V}{\epsilon \sigma S_b T_s^3}\left[\frac{1}{4}\ln\left|\frac{T_b + T_s}{T_b - T_s}\right| + \frac{1}{2}\tan^{-1}\left(\frac{T_b}{T_s}\right) - \frac{\pi}{4}\right] \tag{4}$$

For the special case where $T_s = 0$ and $T_{b,0} = \infty$, the equation becomes

$$t = \frac{\rho c V}{3\epsilon \sigma S_b T_b^3} \tag{5}$$

Equation (4) is plotted in figure 1(a) for a sink temperature of 700° R and the parameter $V/\epsilon S$ equal to 1.0 Btu/(hr)(sq ft/in.)(°R⁴) over a range of values of ρc from 10 to 90 Btu/(cu ft)(°R). The values of the parameters are representative of those encountered in rocket-engine design; for example, values of ρc for steel range from 50 to 60. The curves are repeated in figure 1(b) for a sink temperature of 400° R. Equation (5) is presented in curve form in figure 1(c). The abscissa of figure 1 represents the cooling time from an initial temperature $T_{b,0}$ of infinity to a finite temperature T_b. It is interesting to note that the cooling rate at very high temperatures is very rapid; for example, the cooling time from a very high temperature to 2500° R is less than 0.1 hour for all cases presented. These curves can also be utilized to find cooling times given by equation (3) (for $T_{b,0}$ finite) by choosing values of time corresponding to both $T_{b,0}$ and T_b. If these values are subtracted, the difference corresponds to the net cooling time between the two temperatures. For example, in figure 1(a), if the cooling time between an initial temperature $T_{b,0} = 2000°$ R and a final temperature $T_b = 780°$ R is desired for a value of $\rho c = 50$, the time t_c corresponding to $T_{b,0}$ is 0.1 hour, and the time corresponding to T_b is 2.5 hours. The cooling time between these temperatures is the difference in times, or 2.4 hours.

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As stated previously, the heat dissipation by means of radiation is very rapid at elevated temperatures (above 2000° R). In contrast, the rate of dissipation is greatly reduced at the lower temperatures. In some cases the cooling time may amount to several hours (fig. 1). Equation (3) manifests the parameters that can be manipulated in order to minimize the cooling time. For example, the cooling time will be reduced if the material is chosen so that the product of the material density ρ and specific heat c is small. In addition, the volume - surface-area ratio of the body should be minimized.

The effect of sink temperature on cooling time is shown in figure 2, where the cooling time is plotted against the temperature of the body for three values of sink temperature. The curve for a sink temperature of 700° R would probably be representative of a body exposed to direct sunlight, and the plot for $T_s = 0$ would be more representative of a vehicle isolated from other celestial bodies and not exposed to the sun's radiation. At relatively high body temperatures, the effect of sink temperature on cooling time is negligible; however, at lower values of body temperature the cooling time increases appreciably with sink temperature.

Most bodies in space are exposed to radiation from the sun at one time or another, and unfortunately the radiation retards cooling. The net heat output of a body in space per unit time can be expressed as

$$\frac{dQ_{b,n}}{dt} = \epsilon_b \sigma T_b^4 S_b - \alpha_b Q_{sun} \tag{6}$$

where $dQ_{b,n}/dt$ is the net heat output of the body per unit time, $\epsilon_b \sigma T_b^4 S_b$ is the energy emitted by the body, and $\alpha_b Q_{sun}$ is the amount of the sun's radiation absorbed by the body. Since the amount of radiation received from the sun is fixed at any time t, desirable characteristics for maximum heat dissipation include good emissive properties ($\epsilon = 1.0$) and poor absorption characteristics ($\alpha = 0$) for high-temperature, solar radiation. The emissivity of a surface is a function of the surface material and surface condition. The absorptivity depends upon both the surface and the character of the incoming radiation (ref. 3). As stated previously, in order to maximize the net heat output per unit time, the surface should be associated with good emissive characteristics and poor absorption ability for high-temperature (sun) radiation. Fortunately, some coatings such as painted surfaces exhibit these qualities and therefore provide a means of controlling the temperature of bodies in space (ref. 4). Values of ϵ/α as high as 40 have been observed. High values of ϵ/α will reduce the cooling time significantly if the initial difference between the sink temperature and body temperature is small. However, this is not usually the case for the rocket engine. The temperature difference is usually 1500° or greater. For these cases the second term of equation (6) ($\alpha_b Q_{sun}$) is negligible compared with the first term ($\epsilon_b \sigma T_b^4 S_b$) regardless of the value of absorptivity α_b.

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APPLICATION OF ANALYSIS

The application of the analysis was directed toward the high-chamber-pressure heat-sink (uncooled) rocket engine. Since the combustion temperature of most propellants exceeds the rocket material temperature limits, the running time of the heat-sink rocket is limited to the time required for the material to reach its temperature limit. Therefore, uncooled motors that are designed to withstand more than one firing require a period of time for cooling before another firing cycle is repeated. Since both the running time and the cooling time are functions of the material properties and the geometry of the rocket, both phases (burning and cooling) of the rocket operating cycle are discussed in this section.

Rocket Geometry and Material Temperature

In order to gain an insight into the cooling characteristics of a rocket operating as a component part of a space ship, the rocket geometry must be established. The rocket geometry evolves from the design technique. The design conditions that are normally specified are (1) thrust, (2) propellants, (3) running time, and (4) fabricating material. A group of designs covering a range of thrust from 100 to 1,000,000 pounds and running time from 5 to 30 seconds was investigated. JP-4 fuel and oxygen at a combustion pressure of 300 pounds per square inch absolute were chosen as the propellants, primarily because the combustion-gas properties were readily available (ref. 5). Graphite was selected as the rocket material because it is capable of withstanding the high temperatures associated with rocket combustion.

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A schematic diagram of a rocket nozzle is shown in figure 3. A table of the design conditions is also included. It was assumed that the combustion chamber of the rocket was not a radiating surface, and therefore it was not included in the design analysis. Preliminary calculations indicated that this assumption was justified in that the heat-transfer rates of the nozzle were much larger than those of the combustion chamber. Combustion-chamber conditions were specified at the entrance to the nozzle (station i). The nozzle throat and exit stations (th and e) are also shown. The nozzles were geometrically similar, both the convergent and divergent sections being conical in shape with cone half-angles of 30° and 15°, respectively. The size of the rockets, of course, varied with thrust. The nozzle was assumed to flow full and the ratio of entrance to throat area was fixed ($A_i/A_{th} = 2.5$) for all designs. Two values of exit area ratio ($A_e/A_{th} = 4.0$ and 20.0) were investigated.

With specifications just discussed and tabulated in figure 3, the inside wall configuration and heat-transfer coefficient were established for all designs. Heat-transfer coefficients along the nozzle were calculated by means of the following equation (ref. 6):

$$h_g = 0.023 \frac{k}{d} Re^{0.8} Pr^{1/3}$$

where h_g is the gas-side heat-transfer coefficient.

The temperature distribution and the nozzle wall thickness (thus the outside wall configuration) required to absorb the heat generated during the combustion process were calculated from the method presented in reference 7, which is a finite-difference solution of the heat-conduction equation for heat flow in the radial direction. The input information required to utilize this technique consists of the inside wall geometry, the material properties k, c, and ρ, the material temperature limit T_w, the heat-transfer coefficient h_g, and the running time t_R.

As stated previously, the outside wall shape was calculated for a range of thrusts (10^2 to 10^6) and running time (5 to 30 sec). The ratio of volume to surface area was then computed from the inside and outside wall configuration. Plots of ratio of volume to surface area against thrust for various running times are shown in figure 4(a) for an exit area ratio of 4.0 and in figure 4(b) for an exit area ratio of 20. Values of volume-area ratio vary from approximately 0.1 to 1.4 over the ranges of running time and thrust investigated. In general, the values of volume-area ratio for the exit area ratio of 4 are appreciably higher than values for the exit area ratio of 20 at the same values of thrust and running time. The volume-area curves for the higher values of running time (15 to 30 sec) were not extended to low values of thrust (100 lb), because the heat-flux rates were such that the limiting wall temperature was exceeded for these conditions. E-472

In figure 5 the ratio of average nozzle temperature to inside wall temperature is plotted against thrust for various running time. The values of temperature ratio range from 0.8 to 1.0. The average nozzle temperature was specified as the initial temperature ($T_{b,0}$) at the beginning of the cooling cycle. The average temperature was based on a numerical average of nine values. Three temperatures were computed, at the nozzle entrance, nozzle throat, and nozzle exit. A mass-averaged temperature probably would have been a more realistic value to use in the computation of cooling time, but it was much more difficult to obtain. Preliminary calculations indicated that the mass-averaged temperature was 5 to 10 percent lower than the numerical averages. In the range of initial material temperatures of this report (3000° R), errors in the initial temperature of as much as 30 percent have a negligible effect on the cooling time (fig. 1).

Cooling Effectiveness

In order to illustrate the effectiveness of radiation cooling, the results of several sample calculations (with design conditions previously described) are presented in curve form. It is assumed that no heat was radiated during the burning time. Since the burning time was very small compared with the cooling time in all cases, the assumption was felt to be justified. The variation of cooling time with various rocket design parameters is shown in figure 6. The emissivity was taken as unity, and the sink temperature T_s and final temperature T_b were specified as 400° and 500° R, respectively. The effect of thrust on cooling time is shown in figure 6(a), where cooling time is plotted against thrust for two nozzle exit area ratios (4 and 20), and a fixed value of running time (20 sec). There is a significant variation of cooling time with thrust. For an exit area ratio of 4, the cooling time varies from 3.5 to 6.0 hours. The cooling times for an area ratio of 20 are considerably reduced, varying from 2 to 3.5 hours over the thrust range.

In figure 6(b) cooling time is plotted against running time for two area ratios and a value of thrust of 1000 pounds. The cooling time varies nearly linearly with running time for both area ratios. Again, considerably reduced cooling times result as the nozzle exit area ratio increases (primarily because of the variation in ratio of volume to surface area). Both examples illustrate that in practical applications radiation cooling is a relatively slow process, in that the cooling times varied from 1 to 7 hours for the cases considered.

In figure 7, cooling time is plotted against total impulse with thrust and running time as parameters. The design conditions are the same as those in figure 6. This plot can be utilized to determine the cooling effectiveness over a wide range of design conditions if both the total impulse required to perform a maneuver and the number of maneuvers per day are known. For example, if it is necessary to operate an uncooled rocket eight times per day on an interplanetary mission, the maximum cooling time between runs is 3 hours. Assume, in addition, that 10 impulse units are expended during the rocket firing. In figure 7, at a value of $I = 10$ and $t_c = 3$, the maximum value of running time is approximately 18 seconds. Since impulse is the product of running time and thrust, the resultant thrust is 2000 pounds. Running times below this figure can be chosen. However, a higher value of thrust must be used. If a lower value of running time is desired (e.g., 10 sec), the thrust will be 3600 pounds and the cooling time 1.5 hours. It should be noted that cooling time will change as the design parameters are varied. As stated previously, the cooling time will increase as emissivity decreases; and, for a fixed sink temperature, cooling time will increase as the final temperature approaches the sink temperature (eq. (3)).

SUMMARY OF RESULTS

An analysis of the cooling characteristics of a heated body in space has been presented. The following results were obtained from the analysis:

1. As shown in equation form, the cooling time of a body from one temperature to another in space is directly proportional to the volume-area ratio of the body, the density of the body, and the specific heat of the body. The cooling time is also a function of the temperature of the surroundings and the emissivity.

2. The cooling time is insensitive to the initial temperature above temperature of 2000° R.

The results of the analysis were applied to an uncooled rocket engine with a chamber pressure of 300 pounds per square inch operating in space, and the following results were obtained:

1. For the range of running times considered (5 to 30 sec) the cooling time of the rocket varied approximately linearly with running time.

2. The cooling time was decreased significantly as the exit area ratio of the rocket increased.

3. For most practical applications, frequent operation of uncooled rockets (at intervals of 1 hr or less) is impractical because of the cooling time involved.

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APPENDIX A

SYMBOLS

A	cross-sectional area, sq ft
B_{bb}	energy emitted by body that is ultimately reabsorbed by body
C	constant of integration
c	specific heat, Btu/(lb)(^{o}R)
d	nozzle diameter, ft
F	thrust, lb
h	convective heat-transfer coefficient, Btu/(hr)(sq ft)(^{o}R)
I	impulse, lb-hr
k	thermal conductivity, Btu/(hr)(ft)(^{o}R)
Pr	Prandtl number
Q	heat output, Btu
Re	Reynolds number
r	radius, ft
S	surface area, sq ft
T	temperature, ^{o}R
t	time, hr
V	volume, cu ft
W	weight, lb
α	absorptivity
β	cone angle, deg
ϵ	emissivity

ρ	density, lb/cu ft
σ	Stefan-Boltzmann constant, 0.173×10^{-8} Btu/(hr)(sq ft)($^{\circ}R^4$)

Subscripts:

av	average
b	body
c	cooling
e	exit
g	gas
i	inlet
in	inside
n	net
out	outside
R	burning
s	sink
sun	sun
th	throat
w	wall
θ	celestial body
0	initial time equal to zero

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APPENDIX B

DERIVATION OF EQUATION FOR COOLING CHARACTERISTICS OF HEATED BODY IN SPACE

The radiant energy leaving a body in space $dQ_{b,n}/dt$ can be expressed as

$$\frac{dQ_{b,n}}{dt} = \frac{dQ_b}{dt} - \frac{dQ_b}{dt} B_{bb} - \frac{dQ_{sun}}{dt} - \frac{dQ_\theta}{dt} \tag{B1}$$

where dQ_b/dt is the energy emitted by the body, B_{bb} is the fraction of energy emitted by the body that is ultimately reabsorbed by the body, dQ_{sun}/dt is the energy received from the sun, and dQ_θ/dt is the energy received from other celestial bodies.

With the assumption that the body is "gray" and the energy emitted by the body is never reabsorbed by the body,

$$\left.\begin{aligned} \frac{dQ_{sun}}{dt} + \frac{dQ_\theta}{dt} &= \epsilon_b \sigma T_s^4 S_b \\ \frac{dQ_b}{dt} &= \epsilon_b \sigma T_b^4 \\ \frac{dQ_b}{dt} B_{bb} &= 0 \end{aligned}\right\} \tag{B2}$$

Then, equation (B1) becomes

$$\frac{dQ_{b,n}}{dt} = \epsilon_b \sigma T_b^4 S_b - \epsilon_b \sigma T_s^4 S_b = \epsilon_b \sigma S_b (T_b^4 - T_s^4) \tag{2}$$

Equation (2) is the well-known radiation equation. The energy equation can be expressed as follows:

$$\frac{dQ_{b,n}}{dt} = -W_b c \frac{dT_b}{dt} = -\rho S_b \frac{V}{S_b} c \frac{dT_b}{dt} \tag{1}$$

Equating equations (1) and (2) and rearranging terms give

$$dt = \frac{\rho c V}{\epsilon \sigma S_b} \frac{dT_b}{T_s^4 - T_b^4} \tag{B3}$$

or

$$\int dt = \frac{\rho c V}{\epsilon \sigma S_b} \int \frac{dT_b}{T_s^4 - T_b^4} \tag{B4}$$

Integrating (B4) gives

$$t = \frac{\rho c V}{\epsilon \sigma S_b T_s^3} \left[\frac{1}{4} \ln \left| \frac{T_b + T_s}{T_b - T_s} \right| + \frac{1}{2} \tan^{-1}\left(\frac{T_b}{T_s}\right) \right] + C \tag{B5}$$

In order to evaluate the constant of integration C, the following boundary condition was utilized: When $t = 0$, $T_b = T_{b,0}$. Equation (B5) then becomes

$$t = \frac{\rho c V}{\epsilon \sigma S_b T_s^3} \left[\frac{1}{4} \ln \left| \frac{T_b + T_s}{T_b - T_s} \right| + \frac{1}{2} \tan^{-1}\left(\frac{T_b}{T_s}\right) - \frac{1}{4} \ln \left| \frac{T_{b,0} + T_s}{T_{b,0} - T_s} \right| - \frac{1}{2} \tan^{-1}\left(\frac{T_{b,0}}{T_s}\right) \right] \tag{3}$$

Equation (3) expresses the cooling time of a body in space between any two temperatures $T_{b,0}$ and T_b. If $T_{b,0}$ is large compared with T_s, then

$$\left(\frac{1}{4} \ln \left| \frac{T_{b,0} + T_s}{T_{b,0} - T_s} \right| + \frac{1}{2} \tan^{-1} \frac{T_{b,0}}{T_s} \right) \cong \frac{\pi}{4}$$

and equation (3) for this condition reduces to

$$t = \frac{\rho c V}{\epsilon \sigma S_b T_s^3} \left[\frac{1}{4} \ln \left| \frac{T_b + T_s}{T_b - T_s} \right| + \frac{1}{2} \tan^{-1}\left(\frac{T_b}{T_s}\right) - \frac{\pi}{4} \right] \tag{4}$$

For the special case of $T_s = 0$, equation (B4) integrates to

$$t = \frac{\rho c V}{3 \epsilon \sigma S_b T_b^3} + C \tag{B6}$$

For the boundary condition $t = 0$, $T_b = T_{b,0}$, equation (B6) becomes

$$t = \frac{\rho c V}{3 \epsilon \sigma S_b} \left(\frac{1}{T_b^3} - \frac{1}{T_{b,0}^3} \right) \tag{B7}$$

and for very large $T_{b,0}$, equation (B7) reduces to

$$t = \frac{\rho c V}{3 \epsilon \sigma S_b T_b^3} \tag{5}$$

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REFERENCES

1. Hast, D. R.: The Ultra-Low Pressure Liquid Propellant Space Propulsion System. TN 58-218, WADC, June 1958.

2. Brown, Aubrey I., and Marco, Salvatore M.: Introduction to Heat Transfer. McGraw-Hill Book Co., Inc., 1951.

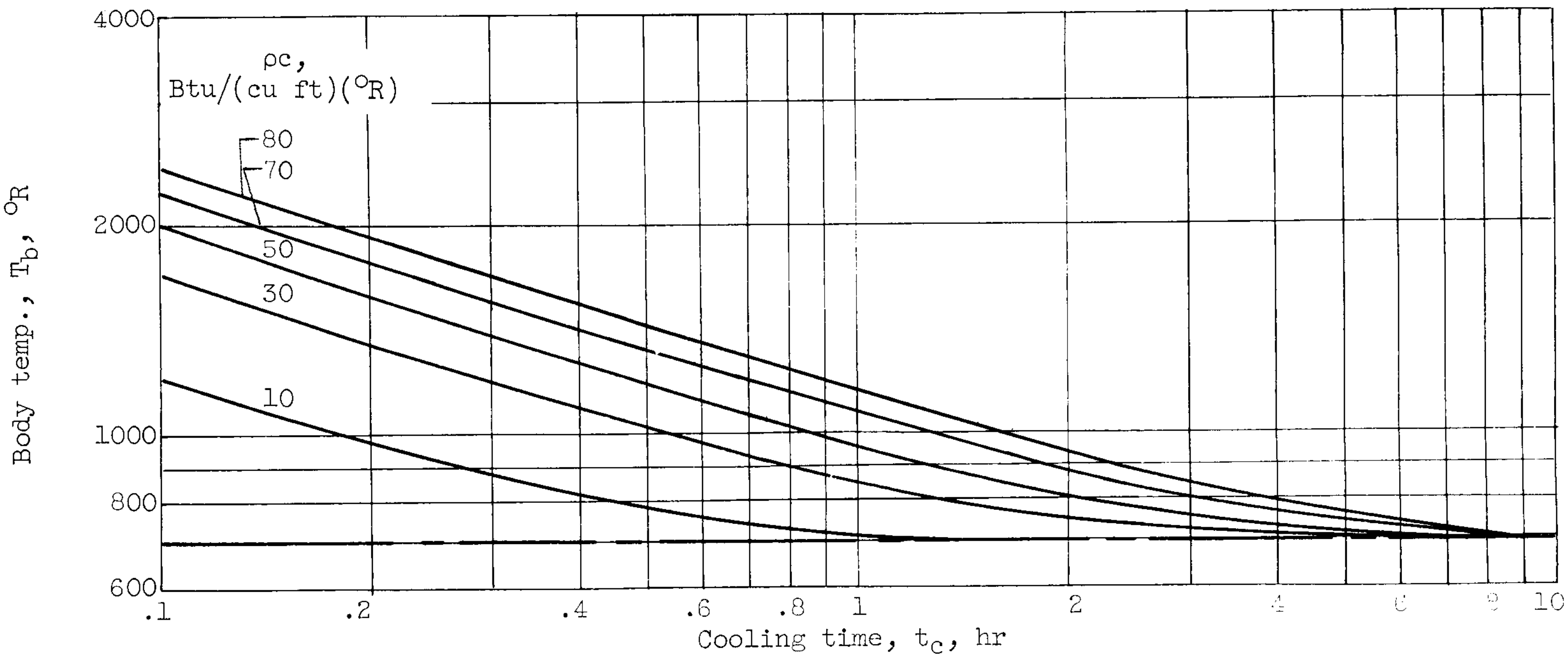
3. Eckert, E. R. G.: Introduction to the Transfer of Heat and Mass. McGraw-Hill Book Co., Inc., 1950.

4. Buwalda, E., and Hibbs, A. R.: Satellite Temperature Measurements for 1958 Alpha-Explorer I. External Pub. 481, Am. Rocket Soc., 1958.

5. Huff, Vearl N., Fortini, Anthony, and Gordon, Sanford: Theoretical Performance of JP-4 Fuel and Liquid Oxygen as a Rocket Propellant. II - Equilibrium Composition. NACA RM E56D23, 1956.

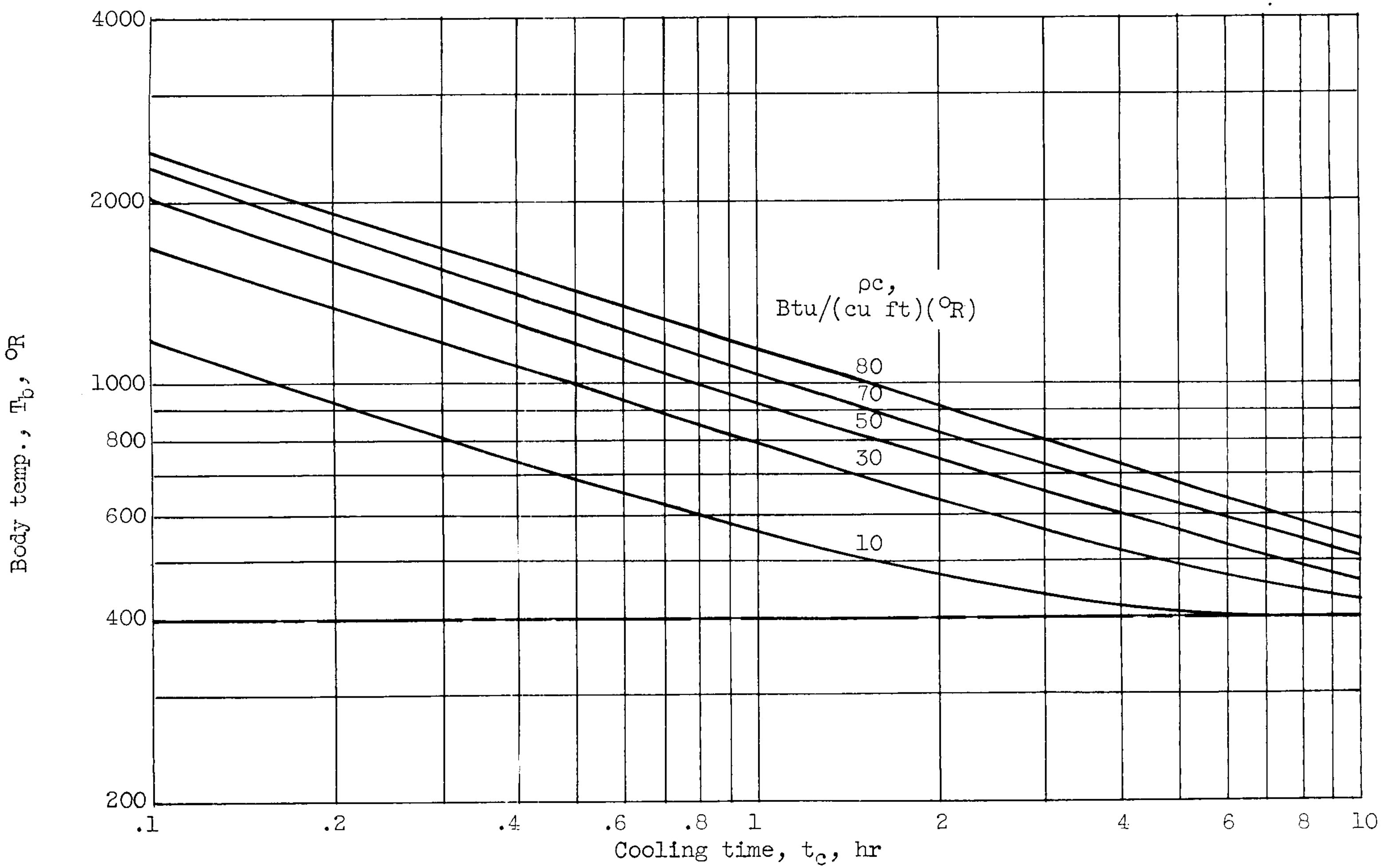
6. McAdams, William H.: Heat Transmission. Second ed., McGraw-Hill Book Co., Inc., 1942.

7. Hatch, James E., Schacht, Ralph L., Albers, Lynn U., and Saper, Paul G.: Graphical Presentation of Difference Solutions for Transient Radial Heat Conduction in Hollow Cylinders with Heat Transfer at the Inner Radius and Finite Slabs with Heat Transfer at One Boundary. NASA TR R-56, 1959.



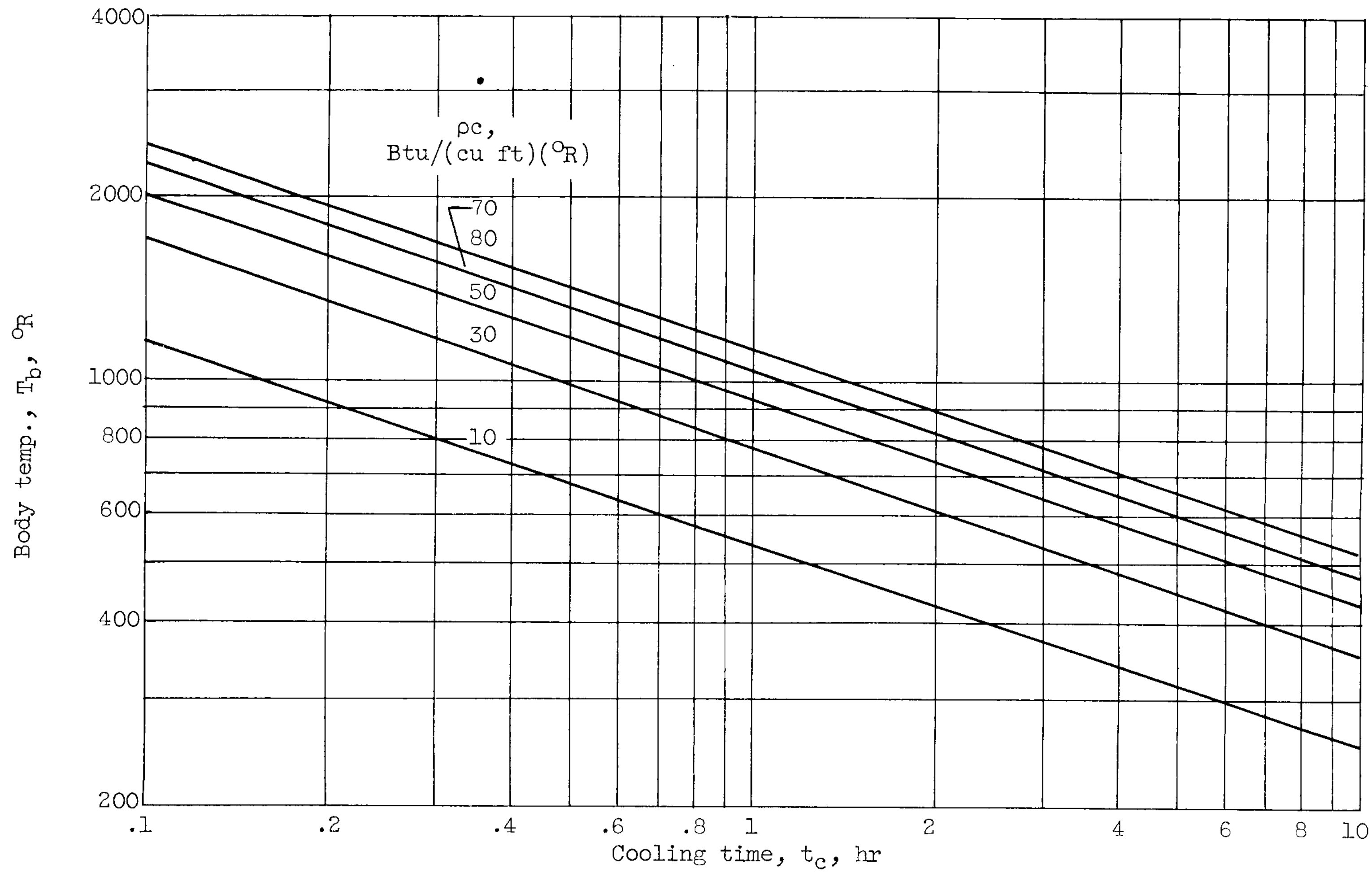
(a) Sink temperature, 700° R.

Figure 1. - Variation of body temperature with cooling time for range of material property values; $V/\epsilon S = 1.0$ Btu/(hr)(sq ft/in.)($^{\circ}R^4$).



(b) Sink temperature, 400° R.

Figure 1. - Continued. Variation of body temperature with cooling time for range of material property values; $V/\epsilon S = 1.0$ Btu/(hr)(sq ft/in.)($^{\circ}R^4$).



(c) Sink temperature, 0^{o} R.

Figure 1. - Concluded. Variation of body temperature with cooling time for range of material property values; $V/\epsilon S = 1.0$ Btu/(hr)(sq ft/in.)($^{o}R^4$).

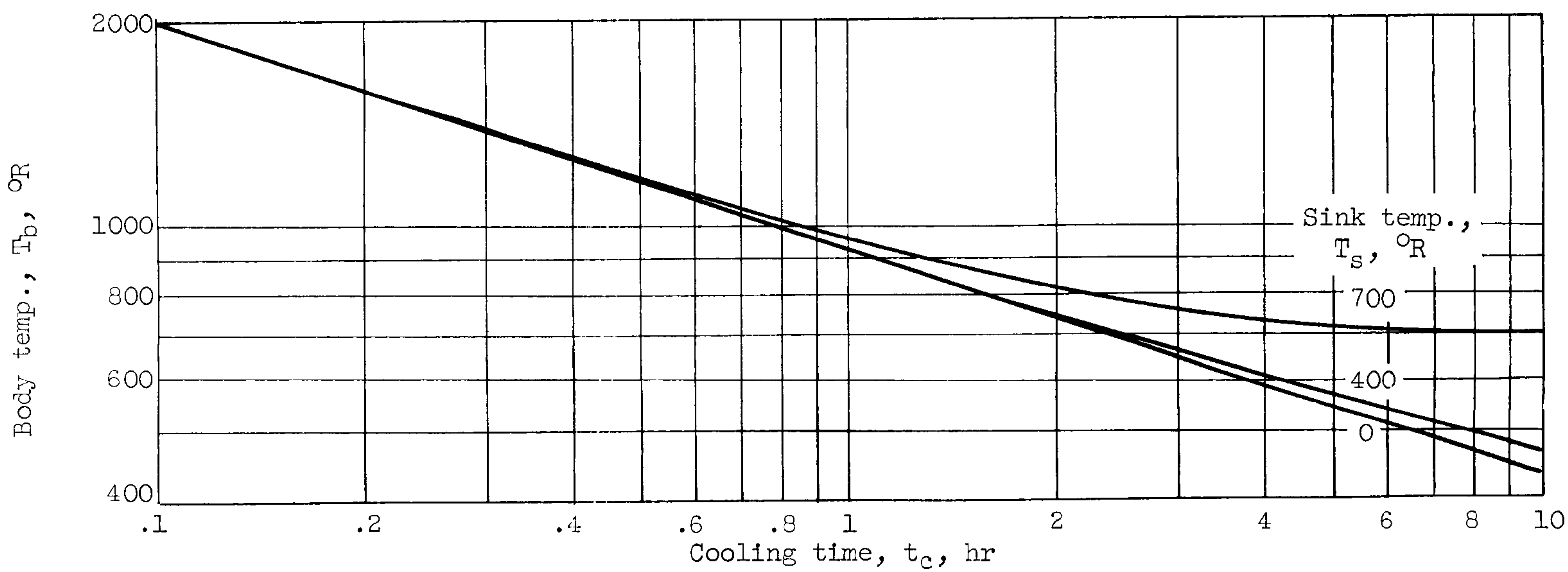
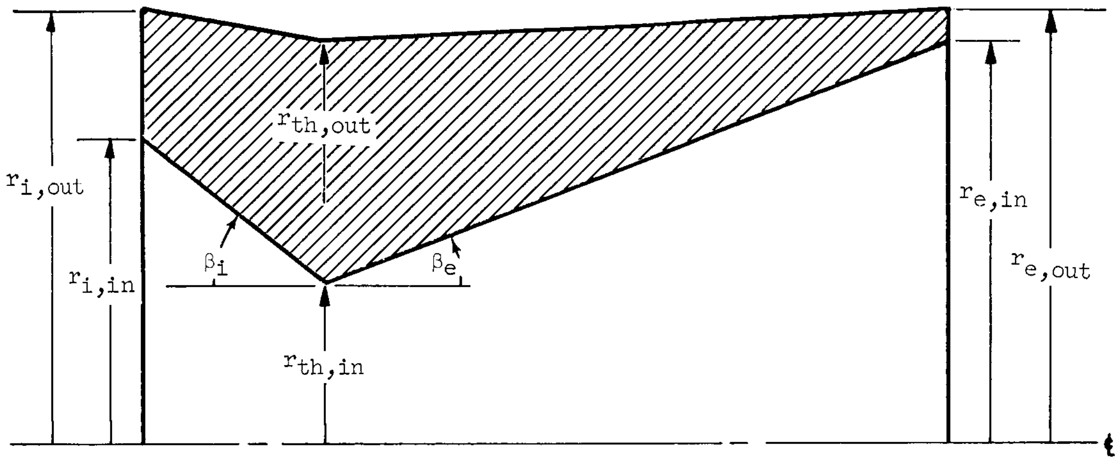


Figure 2. - Effect of sink temperature on cooling characteristics of a body in space; $\rho c = 50$; $V/\epsilon S = 1.0$.



Schematic diagram of rocket nozzle

Design Conditions

Propellants	JP-4 fuel and oxygen
Combustion temperature, $^{\circ}R$	6249
Combustion-chamber pressure, lb/sq in. abs	300
Percent fuel	29.15
Range of thrust, lb	100 to 1,000,000
Range of running time, sec	5 to 30
Material	Graphite
Limiting inside wall temperature, $^{\circ}R$	3500
Specific heat at 1250° R	0.338
Density at 1250° R, lb/cu ft	140
Thermal conductivity	61.4
Diffusivity	1.32
Geometry:	
Nozzle inlet area ratio, A_i/A_{th}	2.5
Nozzle exit area ratio, A_e/A_{th}	4.0 and 20.0
Inlet cone half-angle, deg	30
Exit cone half-angle, deg	15
Adiabatic wall temperature, $^{\circ}R$	5500

Figure 3. - Rocket geometry and design conditions.

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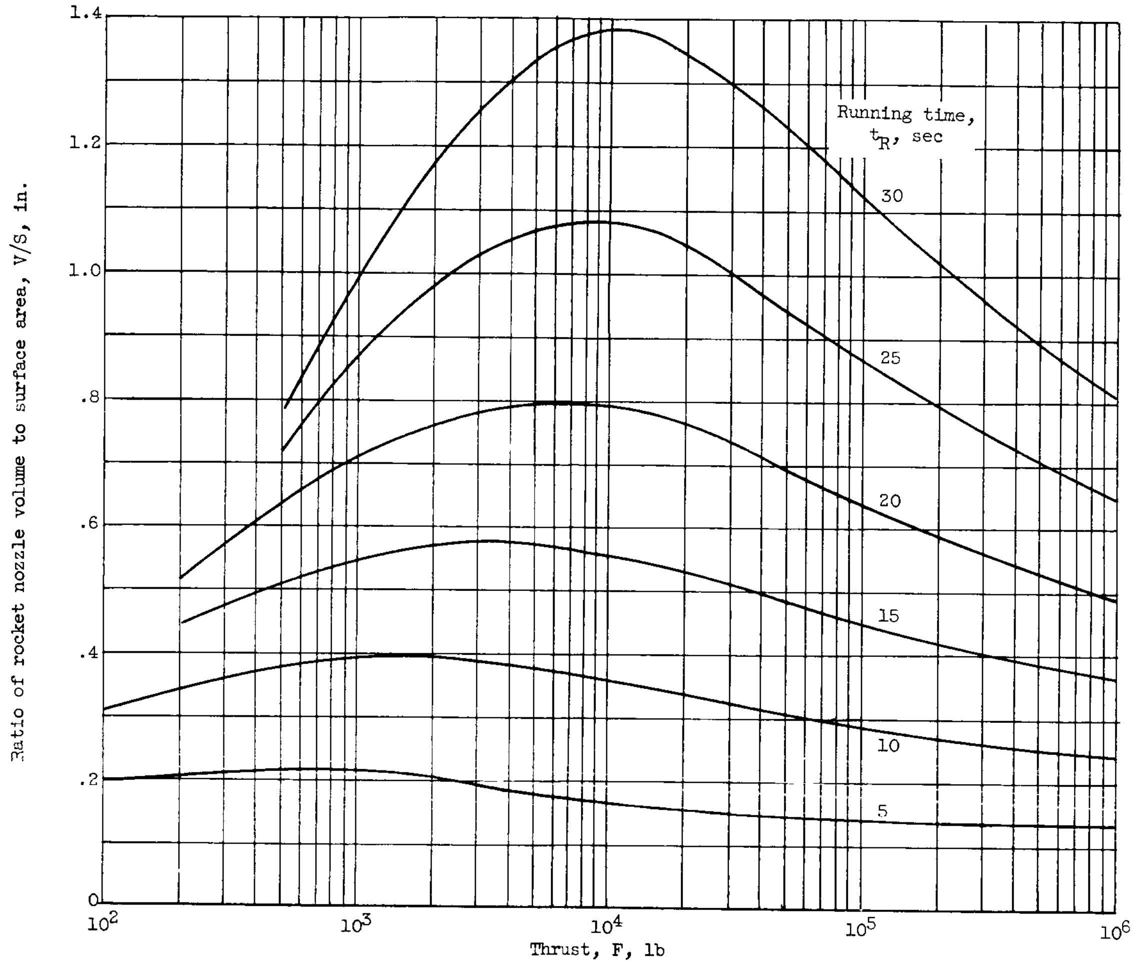
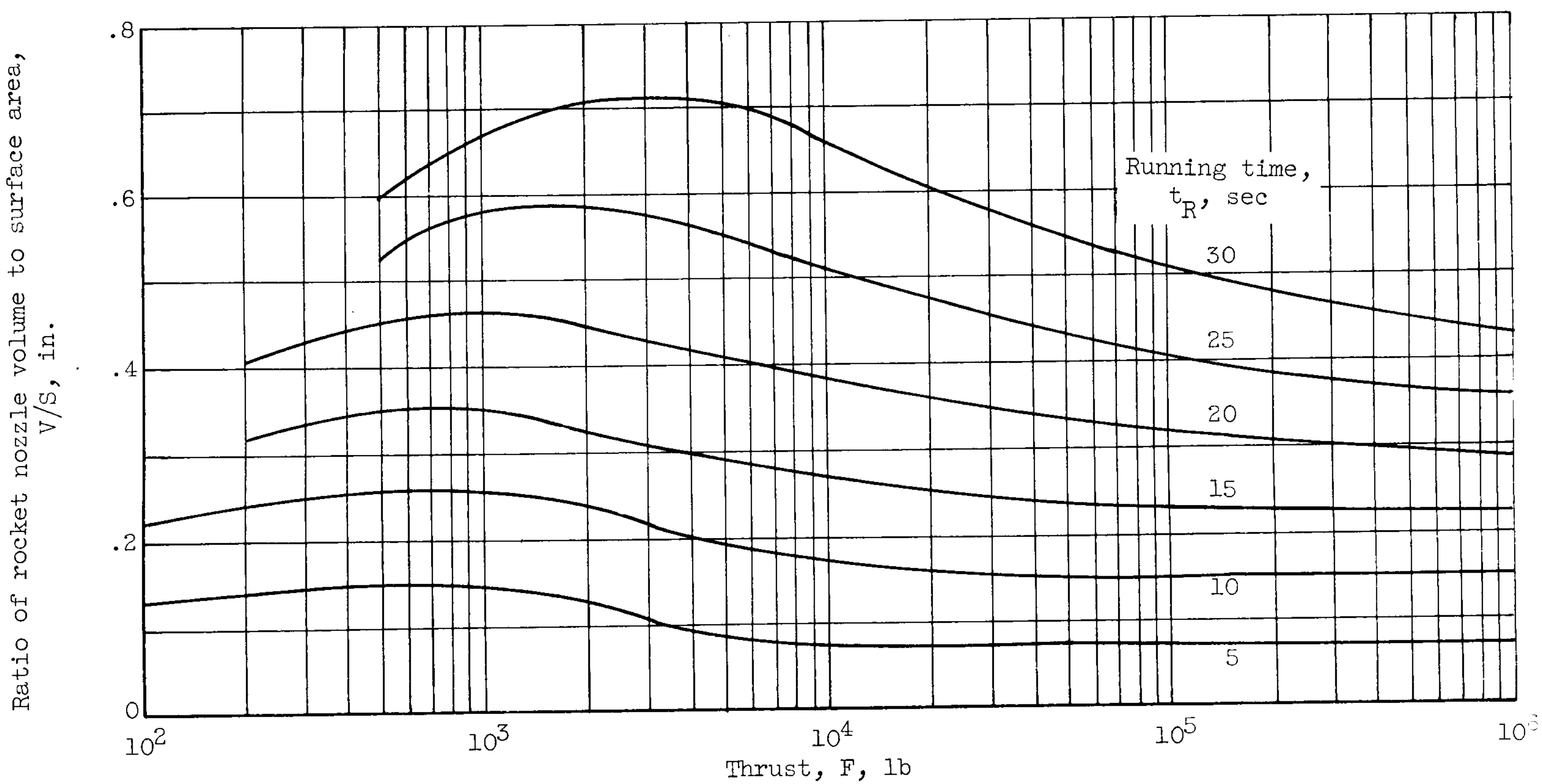


(a) Exit area ratio, A_e/A_{th}, 4.0.

Figure 4. - Variation of rocket nozzle geometry with thrust.



(b) Exit area ratio, A_e/A_{th}, 20.

Figure 4. - Concluded. Variation of rocket nozzle geometry with thrust.

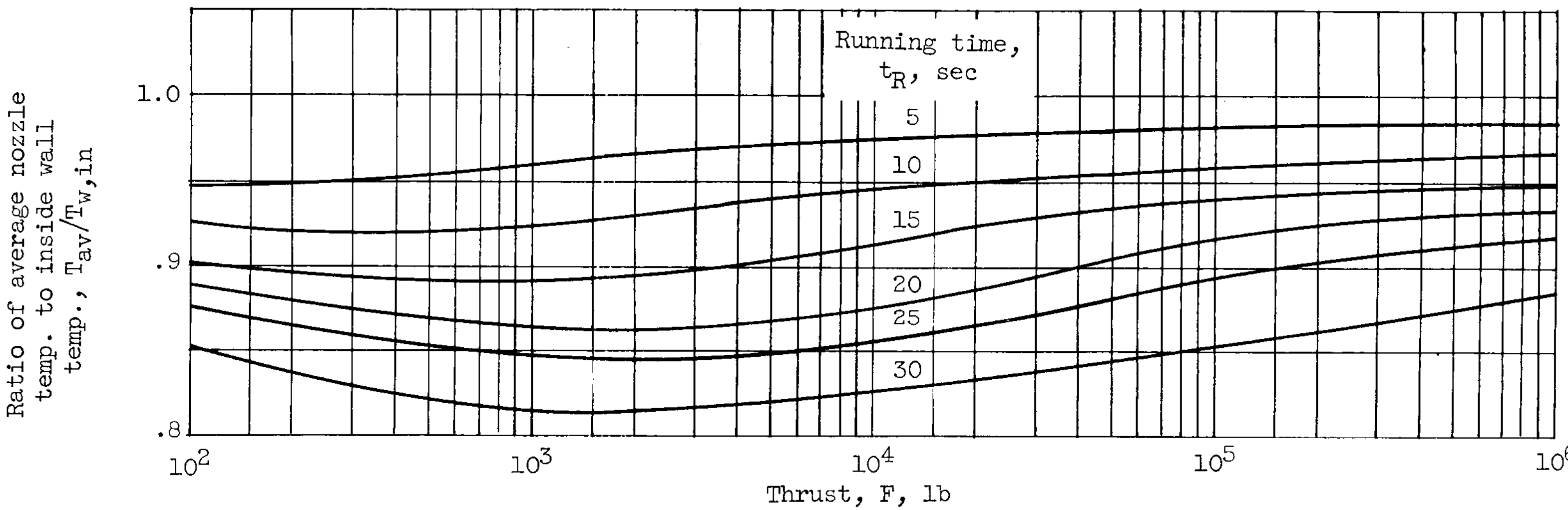
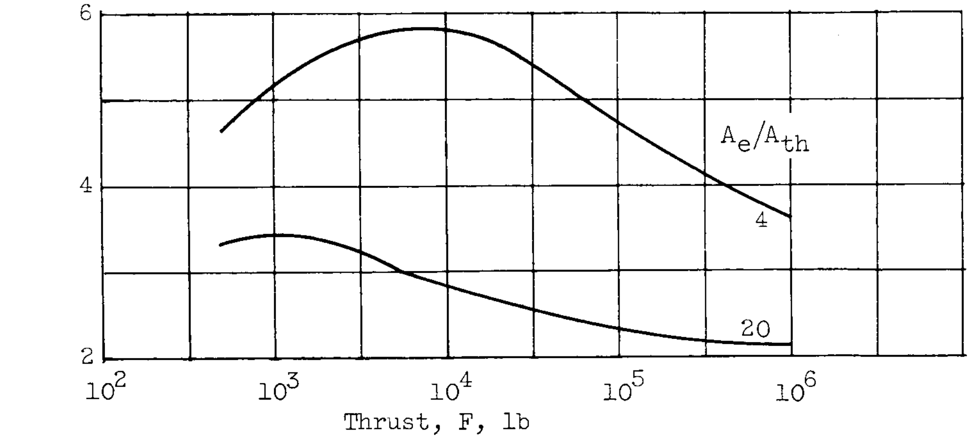
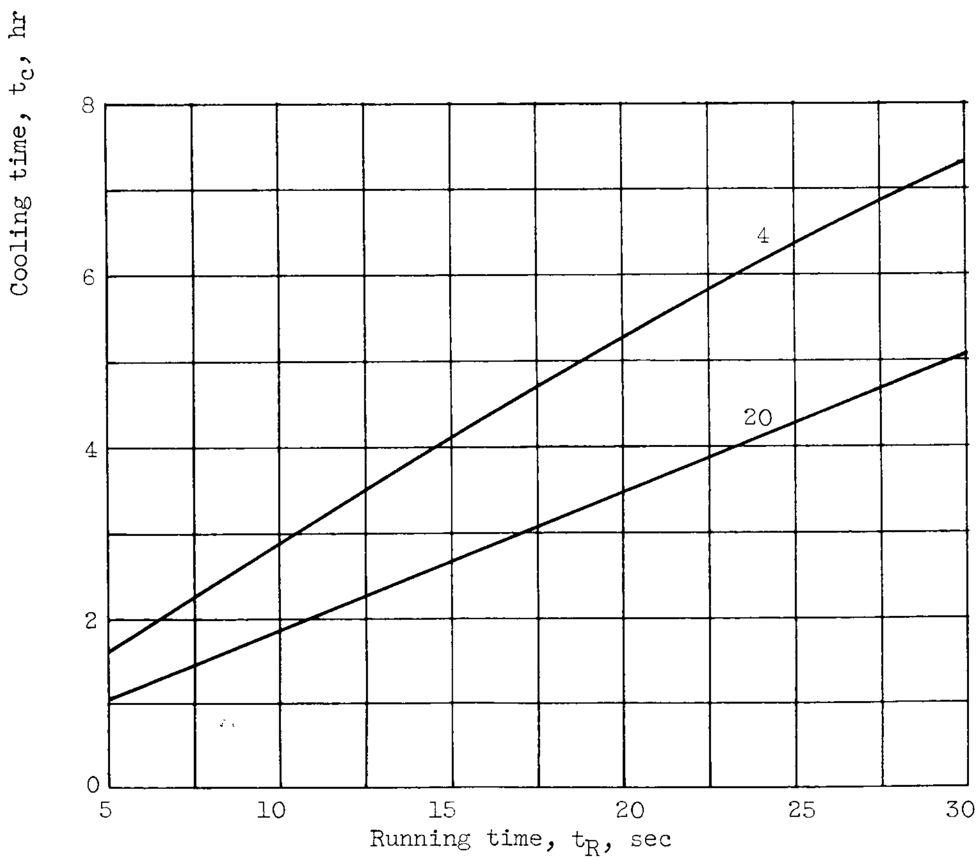


Figure 5. - Variation of ratio of average nozzle material temperature to inside wall temperature with thrust for various running times. Limiting inside wall temperature, $T_{w,in}$, 3500° R.



(a) Effect of thrust. Running time, 20 seconds.



(b) Effect of running time. Thrust, 1000 pounds.

Figure 6. - Variation of cooling time with rocket design parameters. Emissivity, 1.0; sink temperature, 400° R; final body temperature, 500° R.

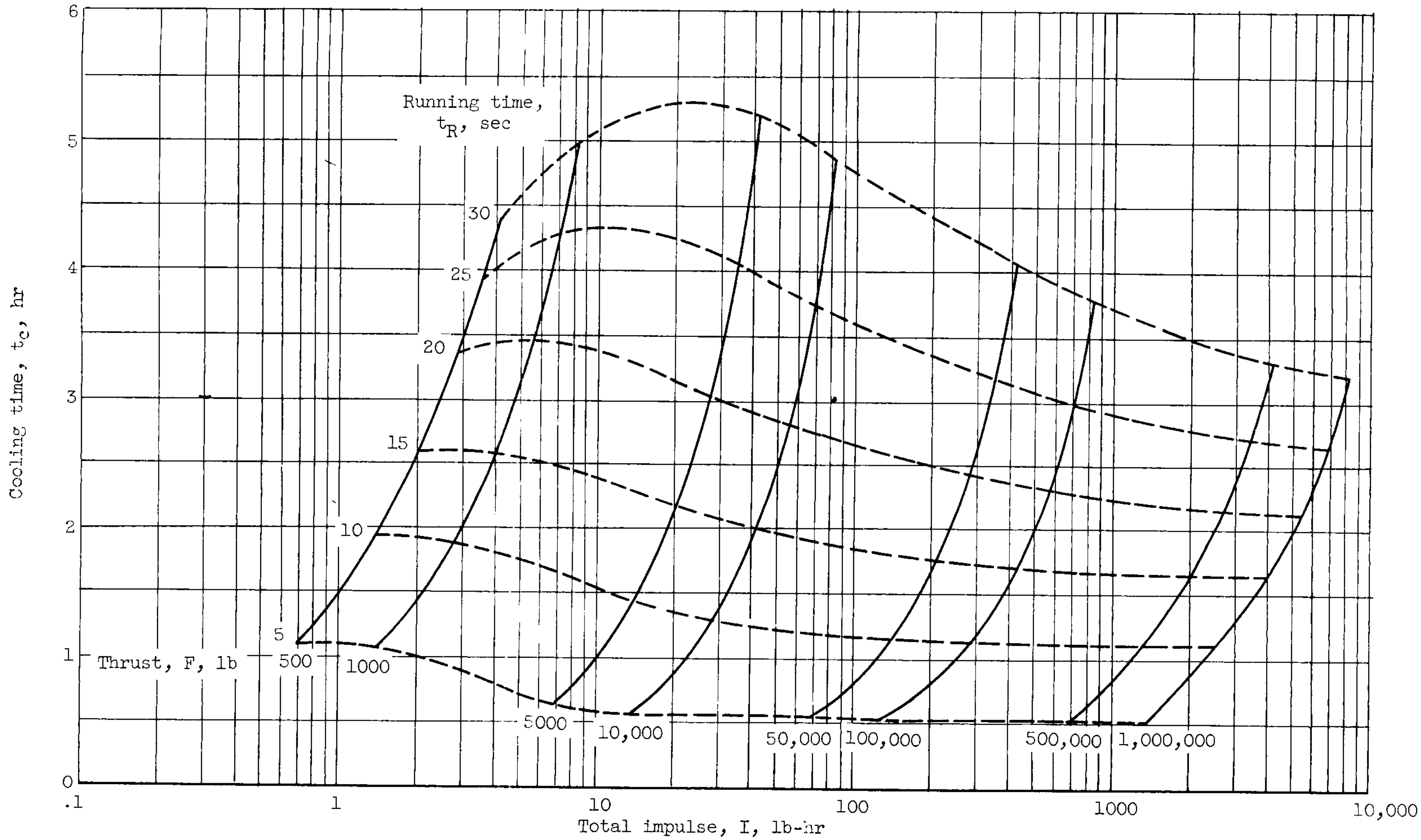


Figure 7. - Variation of cooling time with total impulse for various values of thrust and running time. Sink temperature, 400° R; final body temperature, 500° R; exit area ratio, 20; emissivity, 1.0.